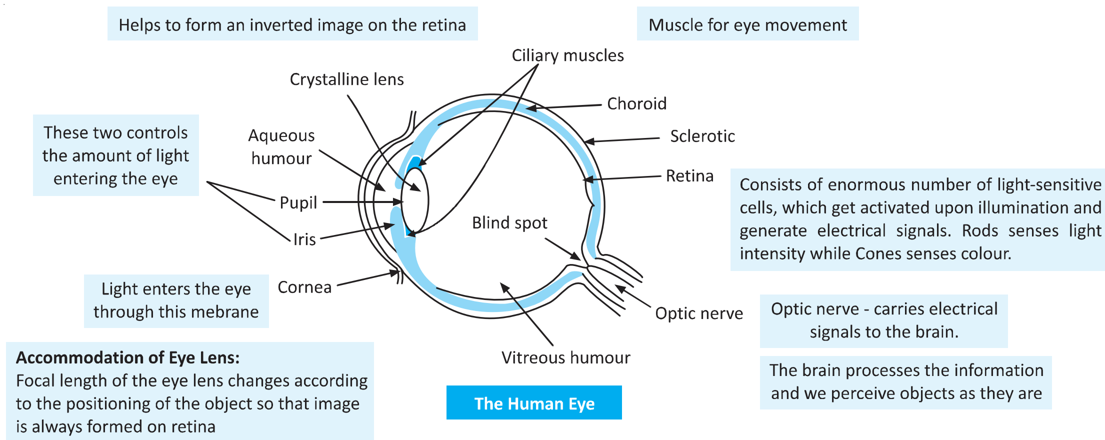
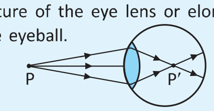
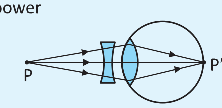
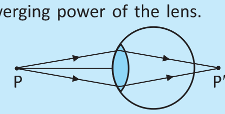
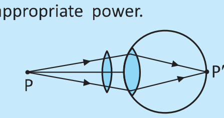
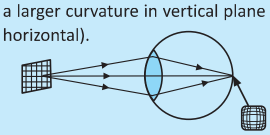
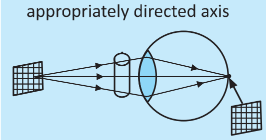
OPTICAL INSTRUMENTS Section - 4

❒ The Human Eye :



The Human Eye

Defects of Vision and their Correction

Defect	Nature	Reason	Correction
Myopia (near-sightedness or short-sight)	Can see nearby objects clearly, but cannot see distant objects distinctly.	The image of a distant object is formed in front of the retina due to excessive curvature of the eye lens or elongation of the eyeball. 	Use concave lens of suitable power 
Hypermetropia (far-sightedness or long-sight)	Can see distant objects distinctly, but cannot see nearby objects clearly.	The image of a nearby object is formed behind the retina due to the decreased converging power of the lens. 	Use a convex lens of appropriate power. 
Presbyopia	The power of accommodation of the eye decreases with ageing. The near point gradually recedes away due to which it becomes difficult to see nearby objects comfortably and distinctly.	Due to the gradual weakening of the ciliary muscles and diminishing flexibility of the eye lens, the power of accommodation of the eye usually decreases with ageing.	Bifocal lenses are used in most cases.
Astigmatism	The image formed on the retina is distorted. A person cannot simultaneously focus on both horizontal and vertical lines.	When the cornea is not spherical in shape, the lines in one direction becomes well focused while those in a perpendicular direction may appear distorted (Example cornea could have a larger curvature in vertical plane than horizontal).  Image as formed on the retina	Use a cylindrical lens of desired radius of curvature with an appropriately directed axis  Image as formed on the retina

Note : The shortest distance for which the lens can focus light on the retina is called least distance of distinct vision OR near point.

❐ Simple Microscope :

Your perception of the size of objects is essentially the angle subtended by them at your eye. For example, a tall building at a great distance from you and a person standing relatively close to you appear to be the same size because they subtend the same angle at your eye.

Therefore, what matters for our perception is not the actual size of the viewed object but its **angular size**.

It is obvious that if we want to observe small objects clearly, we must look at them from a closer distance. To observe any object, the lens inside our eye must focus on it by changing its shape and hence changing its focal length. So, the eye lens must decrease its focal length to focus on closer objects and increase it to focus on far away objects.

But, you feel more eye strain the nearer you need to focus, and objects closer than 25 cm away from the eye are uncomfortable to bring into focus. This distance is known as the **Least Distance of Distinct Vision** and is denoted by *D*. This distance depends on the physiology of the eye, and may be slightly different for each individual, but 25 cm is taken as a standard for the design of optical instruments.

So, if you want to observe an object clearly, you should bring it closer (or move closer to it) but you cannot bring it closer than 25 cm from the eye without straining your eyes in an inadvisable way.

If you want to observe the object clearer than this, you must use an optical instrument.

A **simple microscope** (a magnifying glass) is simply a thin convex that you can place between your eye and the object you are observing. So, looking through the lens, you will now see the image of the object instead of the object itself. The lens is placed such that the object is nearer to the lens than its focus, and so a virtual and erect image is formed.

Magnifying Power

The magnifying power of a simple microscope is the ratio between the angular size of the object when viewed through the instrument (i.e. the angular size of the image), and the angular size of the object when placed at 25 cm from the eye, because this is the largest you can view the object without using the microscope.

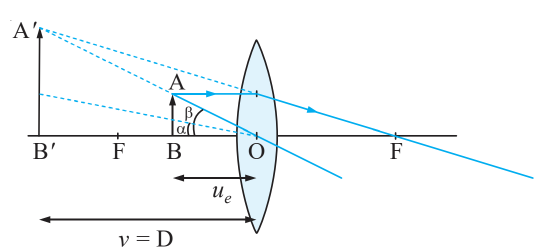
Note that for the following derivations, it is assumed that the eye is very close to the lens (on the right side of the lens in the diagram).

We can move the lens relative to the object such that image is formed anywhere between 25 cm from the eye and infinity. Here, we find the magnifying power in the two extreme cases, known as **Near-Point adjustment** (image formed at 25 cm from the eye, or lens) and **Normal adjustment** (image formed at infinity).

In each case, α is the angular size of the object if it was at 25 cm from the eye

β is the angular size of the image

(i) Near-point Adjustment :



$$\text{M.P.} = \frac{\beta}{\alpha} \approx \frac{\tan\beta}{\tan\alpha} = \frac{A'B'}{B'O} \div \frac{AB}{B'O} \quad ; \quad \text{M.P.} = \frac{A'B'}{AB} = \frac{v}{u} = \frac{-D}{-u_e}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$v = -D$$

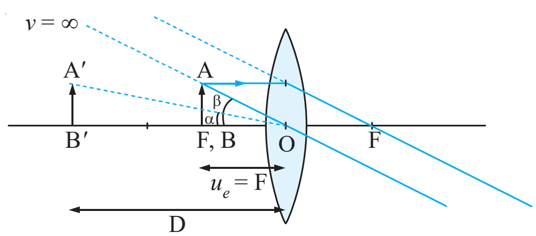
$$u = -u_e$$

$$\frac{1}{F} + \frac{1}{D} = \frac{1}{u_e}$$

$$\text{M.P.} = \frac{D}{u_e} = D\left[\frac{1}{F} + \frac{1}{D}\right]$$

$$\text{M.P.} = \frac{D}{F} + 1$$

(ii) Normal Adjustment :



$$\text{M.P.} = \frac{\beta}{\alpha} \approx \frac{\tan\beta}{\tan\alpha} = \frac{\frac{AB}{F}}{\frac{A'B'}{D}} = \frac{D}{F} \qquad (\because \; A'B' = AB = \text{Object size})$$

Magnifying power in nearby adjustment is more than in normal adjustment.

Magnification of an image by lens and angular magnification (or M.P.) of an optical system are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the image even if the image is magnified) to the angular size of the object if placed at the near point (25 cm). Thus magnification magnitude is $\left|\frac{v}{u}\right|$ and M.P. is $(25/u)$. Only when the image is located at the near point $v = 25\ cm$, are the 2 quantities equal.

Illustration - 46 *A man with a normal near-point (25 cm) wants to read a book with small print using a simple microscope: a thin convex lens of focal length 15 cm by holding it close to his eyes.*

(a) *What is the minimum and maximum separation he can maintain between the book and the lens for comfortable reading? Also find the magnifying power provided by the lens in each case.*

(b) *What separation should he maintain between the book and the lens to get a magnifying power of 2?*

SOLUTION :

(a) For comfortable viewing, the final image can be anywhere between 25 cm away from the eye and infinity. Also, the image should be virtual and erect. So, we know that the distance between the book and the lens must be less than the focal length of the lens, i.e. 15 cm.

When the book is 15 cm away from the lens, the image is formed at infinity.

For the image to be 25 away from the eye, the image should be 25 cm away from the lens, i.e. $v = -25$ cm.

So, $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{15} \Rightarrow u = -\frac{75}{8} = -9.38\ cm$

So, the minimum and maximum separation between the book and the lens are 9.38 cm and 15 cm.

Angular magnification in each case $= \frac{25\ cm}{\text{Distance of the object from the eye}}$

Therefore, in the first case (minimum separation), Magnifying power $= \frac{25}{\left(\frac{75}{8}\right)} = \frac{8}{3} = 2.67$

And, in the second case (maximum separation) Magnifying power $= \frac{25}{15} = 1.67$

(b) We are given that magnifying power = 2

Therefore, distance of the object from the eye should be $\frac{25}{2} = 12.5\ cm$

Hence, he should maintain a separation of 12.5 cm between the book and the lens.

Illustration - 47 *A card sheet divided into squares each of size* 1 mm^2 *is being viewed at a distance of* 9 *cm through a magnifying glass* (*a converging lens of focal length* 10 *cm*) *held close to the eye.*

(a) *What is the magnification produced by the lens? How much is the area of each square in the virtual image?*

(b) *What is the angular magnification* (*magnifying power*) *of the lens?*

(c) *Is the magnification in* (a) *equal to the magnifying power in* (b)? *Explain*

SOLUTION :

(a) $\frac{1}{v} + \frac{1}{9} = \frac{1}{10}$ i.e., $v = -90\ cm$

Magnitude of magnification = 90/9 = 10

Each square in the virtual image has an area $10 \times 10 \times 1\ mm^2 = 100\ mm^2 = 1\ cm^2$

(b) Magnifying power = 25/9 = 2.8

(c) No, magnification of an image by a lens and angular magnification (or magnifying power) of an optical instrument are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the image even if the image is magnified) to the angular size of the object if placed at the near point (25 *cm*). Thus, magnification magnitude is $|(v/u)|$ and magnifying power is $(25/|u|)$. Only when the image is located at the near point $|v| = 25\ cm$, are the two quantities equal.

Illustration - 48 *Answer the following questions*:

(a) *The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?*

(b) *In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?*

(c) *Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?*

SOLUTION :

(a) Even though the absolute image size is bigger than the object size, the angular size of the image is equal to the angular size of the object. The magnifier helps in the following way: without it object would be placed no closer than 25 *cm*; with it the object can be placed much closer. The closer object has larger angular size than the same object at 25 *cm*. It is in this sense that angular magnification is achieved.

(b) Yes, it decreases a little because the angle subtended at the eye is then slightly less than the angle subtended at the lens. The effect is negligible if the image is at a very large distance away. [Note: When the eye is separated from the lens, the angles subtended at the eye by the first object and its image are not equal]

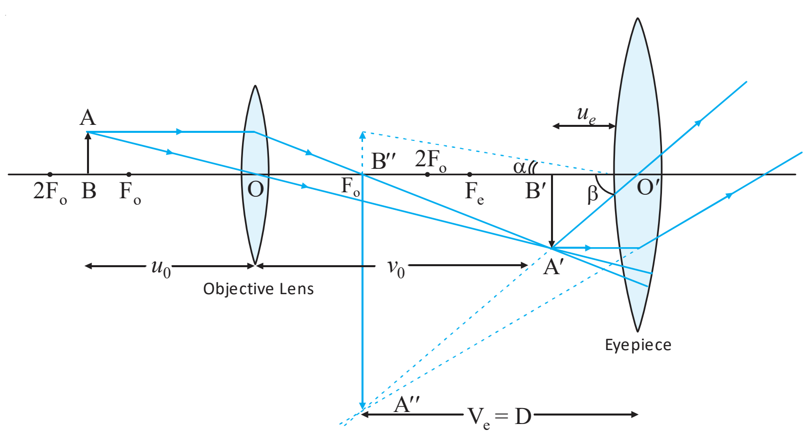
(c) Firstly, fabricating a lens of very small focal length is not easy. More important, if you decrease focal length, aberrations (both spherical and chromatic) become more pronounced. So, in practice, you cannot get a magnifying power of more than 3 or so with a simple convex lens. However, using an aberration corrected lens system, one can increase this limit by a factor of 10 or so.

❐ Compound Microscope :

A simple microscope has a limited maximum magnification for realistic focal lengths. For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a **compound microscope**.

The lens nearest the object, called the **objective**, forms a real, inverted, magnified image of the object. This serves as the object for the second lens, the **eyepiece**, which functions essentially like a simple microscope or magnifier, produces the final image, which is enlarged and virtual. The first inverted image is thus near (at or within) the focus of the eyepiece, at a distance appropriate for final image formation at infinity, or a little closer for image formation at the near point. Clearly, the final image is inverted with respect to the original object.

(i) Near-point Adjustment :



$$L = |v_0| + |v_e|$$

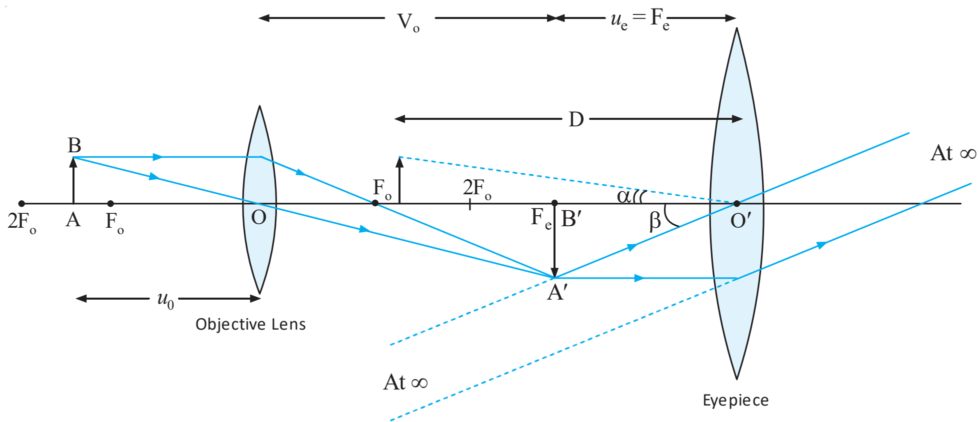
Magnifying Power : Ratio of the angle subtended by the image at the eye to the angle subtended by the object at the eye when both are at least distance of distinct vision from the eye.

$$\text{M.P.} = \frac{\beta}{\alpha} \approx \frac{\tan\beta}{\tan\alpha} = \frac{\frac{A''B''}{B''O'}}{\frac{AB}{B''O'}} = \frac{A''B''}{AB} = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB} = m_e \times m_0$$

Important :

F_o and F_e are small in order to increase the magnifying power. Further F_o is smaller than F_e so that the intermediate image $A'B'$ is formed between O and F_e of the eye piece.

(ii) Normal Adjustment :



Magnifying Power Ratio of the angle subtended by final image formed at infinity at the eye to the angle subtended by the object at the eye when placed at least distance of distinct vision.

$$\text{M.P.} = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{\frac{A'B'}{B'O'}}{\frac{AB}{D}} = \frac{A'B'}{AB} \cdot \frac{D}{B'O'} = \frac{D}{F_e}\left(\frac{-v_0}{u_0}\right) \qquad \text{M.P.} = \frac{-v_0}{u_0} \cdot \frac{D}{F_e}$$

Illustration - 49 *Answer the following questions*:

(a) *Why must both the objective and the eyepiece of a compound microscope have short focal lengths*?

(b) *When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece*?

SOLUTION :

(a) Angular magnification of eye-piece is $[(25/f_e) + 1]$ (f_e in *cm*) which increases if f_e is smaller. Further, magnification of the objective is given by $\frac{v_o}{|u_o|} = \frac{1}{\left(|u_o|/f_o\right) - 1}$ which is large when $|u_o|$ is slightly greater than f_o. The microscope is used for viewing very close object. So $|u_o|$ is small, and so is f_o.

(b) The image of the objective in the eye-piece is known as 'eye-ring'. All the rays from the object refracted by objective go through the eye-ring. Therefore, it is an ideal position for our eyes for viewing. If we place our eyes too close to the eye-piece, we shall not collect much of the light and also reduce our field of view. If we position our eyes on the eye-ring and the area of the pupil of our eye is greater or equal to the area of the eye-ring, our eyes will collect all the light refracted by the objective. The precise location of the eye-ring naturally depends on the separation between the objective and the eye-piece. When you view through a microscope by placing your eyes on one end, the ideal distance between the eyes and eye-piece is usually built-in the design of the instrument.

Illustration - 49 *A compound microscope consists of an objective lens of focal length* 2.0 *cm and an eye-piece of focal length* 6.25 *cm separated by distance of* 15 *cm. How far from the objective should an object be placed in order to obtain the final image at* (a) *the least distance of distinct vision* (25 *cm*)*, and* (b) *at infinity? What is the magnifying power of the microscope in each case* ?

SOLUTION :

Given, focal length of objective lens, $f_O = 2cm$

Focal length of eye-piece $f_e = 6.25\ cm$

Distance between both lenses $L = 15\ cm$

(a) Distance of final image from eye-piece

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{6.25} = \frac{-1-4}{25} = \frac{-5}{25}$$

$$u_e = -5cm$$

As the distance between objective and eye-piece $(v_O + u_e) = 15cm$

Distance of image formed by object lens

$$v_O = L - |u_e| = 15 - 5 = 10\ cm$$

Using the lens equation for objective lens

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} = \frac{4}{10}$$

$$u_o = -2.5cm$$

So, the object should be 2.5 cm in front convex lens. And the magnifying power is :

$$m = \frac{v_o}{u_o}\left(1 + \frac{d}{f_e}\right) = \frac{10}{2.5}\left(1 + \frac{25}{6.25}\right) = 20$$

$$(\because d = 25\ cm)$$

(b) The final image will be formed at infinity only if the image formed by the objective is in the focal plane of the eye-piece i.e., at principal focus of the eye-piece.

Thus, here $v_e = -\infty$, $u_e = f_e = 6.25\ cm$

Image distance of objective lens

$$v_o = L - f_e = -15 - 6.25 = 8.75\ cm$$

Using Lens formula,

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{8.75} - \frac{1}{2} = \frac{2 - 8.75}{17.5}$$

$$u_o = -\frac{17.5}{6.75} = 2.59\,cm$$

So the magnifying power is :

$$m = \frac{v_o}{u_o}\left(1 + \frac{d}{f_e}\right) = \frac{8.75}{2.59}\left(1 + \frac{25}{6.25}\right)$$

$$m = 13.51$$

Illustration - 50 *A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eye-piece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus, with the final image formed at the near-point. What is the separation between the two lenses? Calculate the magnifying power of the microscope ?*

SOLUTION :

Given, focal length of objective $f_e = 8mm = 0.8\ cm$

Focal length of eye – piece $f_e = 2.5\ cm$

Distance of object from objective

$$u_o = -9mm = 0.9\ cm$$

Distance of image from eye – piece ;

$$v_e = -d = -25\ cm$$

Using Lens equation for eye – piece

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

or $$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{2.5}$$

$$= \frac{-1-10}{25} = -\frac{11}{25}$$

Therefore, distance of first image from eye-piece

$$= \frac{25}{11} = 2.27\,cm$$

Using Lens equation for objective

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

or $$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o} = \frac{1}{0.8} - \frac{1}{0.9} = \frac{0.9-0.8}{0.72}$$

Therefore, distance of first image for objective lens $v_o = 7.2\ cm$

Separation between two lenses

$$L = |u_e| + |v_o| = 2.27 + 7.2$$

$$L = 9.47\ cm$$

Magnifying power of compound microscope

$$m = \frac{v_o}{u_o}\left(1 + \frac{d}{f_e}\right) = \frac{7.2}{0.9}\left(1 + \frac{25}{2.5}\right)$$

$$m = 88$$

Illustration - 51 *An angular magnification (magnifying power) of 30 is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5cm. How will you set up the compound microscope?*

SOLUTION :

Assume microscope in normal use i.e., image at 25 cm.

Angular magnification of the eye-piece $= \frac{25}{5} + 1 = 6$

Magnification of the objective $= \frac{30}{6} = 5$.

Therefore $$\frac{V_o}{(-u_o)} - \frac{1}{u_o} = \frac{1}{1.25}$$

$$\frac{1}{-5u_o} - \frac{1}{u_0} = \frac{1}{1.25}$$

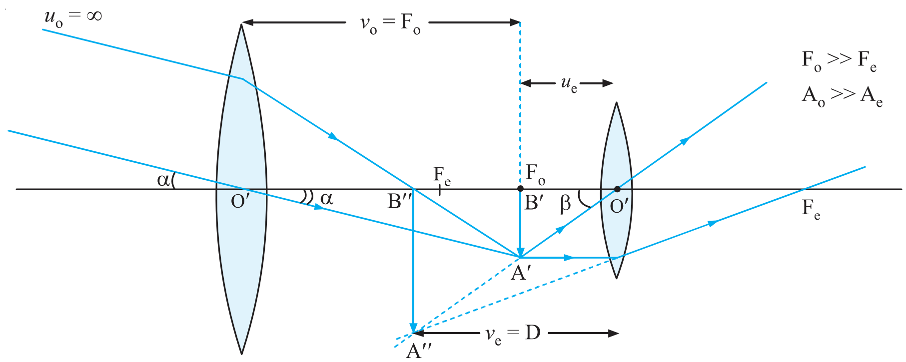
Which gives $u_o = -1.5\ cm$;

$$v_0 = 7.5\ cm\ |u_e| = (25/6)\ cm = 4.17\ cm.$$

The separation between the objective and the eye-piece should but $(7.5 + 4.17)\ cm = 11.67\ cm$. Further the object should be placed $1.5\ cm$ from the objective to obtain the desired magnification.

❒ Refracting Telescope :

A **telescope** is used to provide angular magnification of distant objects. A type of telescope that uses two lenses, an objective and an eyepiece, is known as a **refracting telescope**. But here, the objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image, producing a final inverted image.



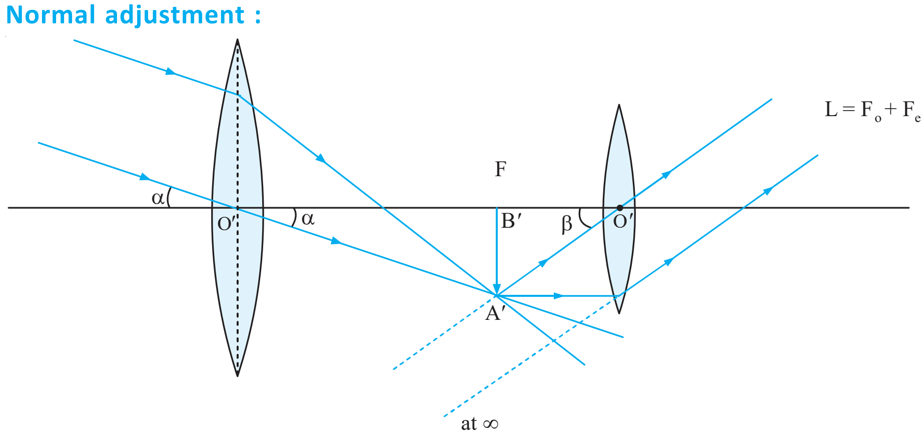
➤ Nearby adjustment :

M.P. It is ratio of angle subtended by the final image formed at least distance of distinct vision on the eye to the angle subtended by the object at the eye when seen directly.

$$\text{M.P.} = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{\frac{A'B'}{B'O'}}{\frac{A'B'}{B'O}} = \frac{B'O}{B'O'} = \frac{F_o}{-u_e} \qquad \left[\begin{array}{l} \frac{1}{F_e} = \frac{1}{v_e} - \frac{1}{u_e} \Rightarrow \frac{1}{F_e} = \frac{-1}{D} + \frac{1}{u_e} \\ \frac{1}{F_e} + \frac{1}{D} = \frac{1}{u_e} \end{array}\right]$$

$$= -F_o\left[\frac{1}{F_e} + \frac{1}{D}\right] = -\frac{F_o}{F_e}\left[1 + \frac{F_e}{D}\right]$$

➤ Normal adjustment :



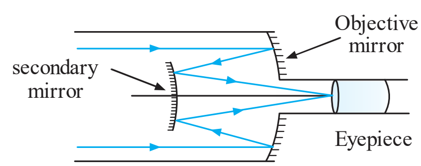
M.P. = $= \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha}$ at $\infty = \frac{\frac{A'B'}{B'O'}}{\frac{A'B'}{B'O'}} = \frac{B'O}{B'O'} = \frac{-F_o}{F_e} = \frac{-F_o}{F_e}$

❒ Reflecting Telescope :

The main considerations with an astronomical telescope are its light gathering power and its resolution or resolving power. The former clearly depends on the area of the objective. With larger diameters, fainter objects can be observed. The resolving power, or the ability to observe two objects distinctly, which are in very nearly in the same direction, also depends on the diameter of the objective. So, the desirable aim in optical telescopes is to make them with objective of large diameter.

Such big lenses tend to be very heavy and therefore difficult to make and support by their edges. Further, it is rather difficult and expensive to make such large sized lenses which form images that are free from any kind of chromatic aberration and distortions.

For these a reasons, modern telescopes use a concave mirror rather than a lens for the objective. Telescopes with mirror objectives are called *reflecting* telescopes.



Schematic diagram of a reflecting telescope (Cassegrain)

Advantages of Reflecting telescopes :

- There is no chromatic aberration in a mirror.
- If a parabolic reflecting surface is chosen, spherical aberration is also removed.
- Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be support over its entire back surface, not just over its rim.

Disadvantage of Reflecting Telescope :

One obvious problem with a reflecting telescope is that the objective mirror focuses light inside the telescope tube. One must have an eyepiece and the observer right there, obstructing some light (depending on the size of the observer cage).

Illustration - 52 *A small telescope has an objective lens of focal length* 144 *cm and an eye-piece of focal length* 6.0 *cm. What is the magnifying power of the telescope ? What is the separation between the objective and the eye-piece*?

SOLUTION :

Given, focal length of objective lens $f_o = 144\ cm$

Focal length of eye-piece $f_e = 6 cm$

Magnifying power of the telescope in normal adjustment (i.e., when the final image is formed at ∞)

$$m = -\frac{f_o}{f_e} = -\frac{144}{6} = -24$$

$\therefore$ Separation between lenses

$$= L = f_o + f_e = 144 + 6 = 150\ cm$$

Illustration - 53 *Answer the following questions:*

(a) *A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eye-piece of focal length 1.0 cm is used, what is the angular magnification of the telescope?*

(b) *If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m, and the radius of lunar orbit is 3.8×10^8 m.*

SOLUTION :

Given, focal length of objective lens, $f_o = 15\ m$

Focal length of eye – piece $f_e = 1\ cm = 0.01\ m$

(a) Angular magnification the telescope

$$m = \frac{f_o}{f_e} = \frac{15}{0.01} = 1500$$

(b) Let d_i be the diameter of the image of the moon formed by the objective lens

Therefore, the angle subtended at the objective lens by the image $= \frac{d_i}{f_O} = \frac{d_i}{15}$

Diameter of object $d_o = 3.48 \times 10^6\ m$

Radius of orbit $r = 3.8 \times 10^8\ m$

The angle subtended by the diameter of the moon

$$= \frac{\text{Diameter of moon}}{\text{Radius of lunar orbit}} = \frac{3.48 \times 10^6}{3.8 \times 10^8}$$

The angle subtended by the image is equal to the angle subtended by the object.

$$\therefore \quad \frac{d_i}{15} = \frac{3.48 \times 10^6}{3.8 \times 10^8}$$

or $\quad d_i = \frac{3.48\ 15 \times 10^{-2}}{3.8} = 13.73 \times 10^{-2}\ m$

or $\quad d_i = 13.73\ cm$

Thus, the diameter of the image of moon is 13.73 *cm*.

Illustration - 54 *A small telescope has an objective lens of focal length 140 cm an eye-piece of focal length 5.0 cm. What is the magnifying power of the telescope for the viewing distance objects when*

(a) *The telescope is in normal adjustment (i.e., when the final image is at infinity)?*

(b) *The final image is formed at the least distance of distinct vision (25 cm)?*

SOLUTION :

Given, focal length of objective lens $f_o = 140\ cm$ and focal length of eye lens $f_e = 5\ cm$

(a) For normal adjustment, the magnifying power

$$m = -\frac{f_o}{f_e} = -\frac{140}{5} = -28$$

(b) For least distance of distinct vision, the magnifying power

$$m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{d}\right) = \frac{140}{5}\left(1 + \frac{5}{25}\right)$$

$$\Rightarrow \quad m = 28(1 + 0.2) = 33.6$$

Illustration - 55 *Answer the following questions:*

(a) *For the telescope described in the previous illustration, what is the separation between the objective lens and the eyepiece?*

(b) *If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?*

(c) *What is the height of the final image of the tower if it is formed at 25cm?*

SOLUTION :

(a) Separation between the lenses $= f_o + f_e = 145\ cm$

(b) Let the height of the image formed by the objective lens be h_o

Angle subtended by the tower at the objective lens

$$= \frac{100\ m}{3\ km} = \frac{1}{30}\ rad$$

Angle subtended by the image formed by the objective lens at the objective lens $= \frac{h_o}{f_o} = \frac{h_o}{140}$

Since the two angles are equal,

$$\frac{h_o}{140} = \frac{1}{30}$$

$$\Rightarrow \quad h_o = \frac{14}{3} = 4.67\ cm$$

(c) For the eye-piece, let the object distance be u

And the image distance, $v = -25\ cm$

Therefore, $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{5}$

$$\Rightarrow \quad u = -\frac{25}{6}\ cm$$

So, magnification produced by the eye-piece,

$$m_e = \frac{v}{u} = \frac{-25}{\left(-\frac{25}{6}\right)} = 6$$

Therefore, height of the final image formed by the eye-piece $h_e = m_e h_o = 6\left(\frac{14}{3}\right) = 28\ cm$

Illustration - 56 *A Cassegrain telescope uses two mirrors. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?*

SOLUTION :

The image formed by the larger (concave) mirror acts as virtual object for the smaller (convex) mirror. Parallel rays coming from the object at infinity will focus at a distance of 110 *mm* from the larger mirror. The distance of virtual object for the smaller mirror = (110 – 20) = 90 *mm*. The focal length of smaller mirror is 70 *mm*.

Using mirror formula for the smaller mirror,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{70} - \frac{1}{90}$$

$$\Rightarrow \quad v = +315\ mm$$

So, the final image is formed 315 mm behind the smaller (convex) mirror.

❒ Resolving Power:

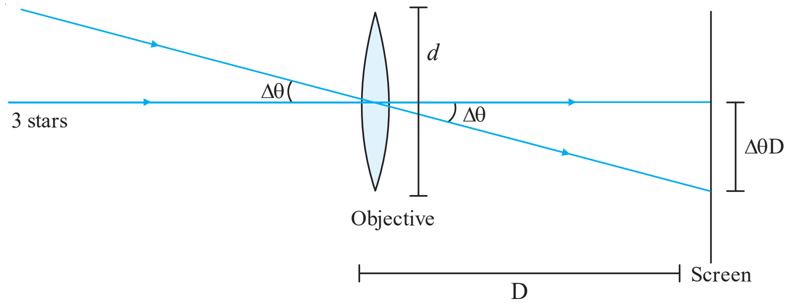
1. General Facts on Resolving Power :

(i) Due to diffraction effects, the images of two distinct points may seem to converge when viewed from a lens. The two images are said to be just resolvable, if the central maxima of one is found to exactly coincide with the first minima of the other. This criteria for resolution is called the Rayleigh's criterion.

(ii) We say that 2 objects are just resolved by a telescope/microscope if the distance between the centre of the images of the 2 objects is at least r_0 (central maxima's should not exactly coincide).

(iii) The angular resolution of the telescope/microscope is determined by the objective lens. The primary purpose of the eye piece is to provide magnification of the image produced by the objective.

2. Resolving Power of Telescope :



For the 2 stars to be just resolved.

$$D\Delta\theta \geq \frac{1.22\,D\lambda}{d} \quad \Rightarrow \quad \Delta\theta_{\min} = \frac{1.22\lambda}{d}$$

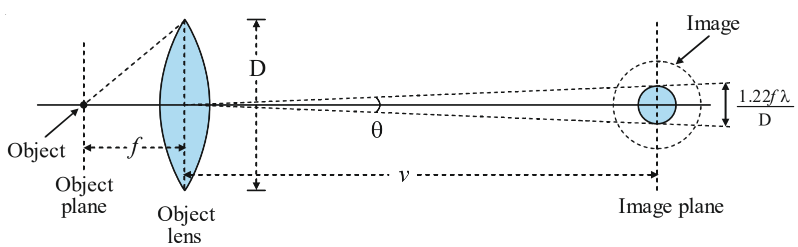
$$\text{Resolving Power} = \frac{1}{\Delta\theta_{\min}} = \frac{d}{1.22\lambda}$$

Where $d = 2a$ if a = radius of lens.

∴ Greater the Resolving power, 2 stars subtending lesser $\Delta\theta$ can be resolved.

We can the resolving power.

3. Resolving Power of Microscope :



Here object are placed at F or very close to F $\therefore u = F$; v = Image Distance

∴ For the 2 objects to be just resolved.

$$v\Delta\theta \approx \frac{1.22\,v\lambda}{D}$$

Two objects with images closer than this distance won't be resolved and will be seen as one.

This minimum separation in object plane is denoted as d_{min}.

Also: $\Delta\theta = \dfrac{d_{\min}}{F} \quad \Rightarrow \quad \dfrac{d_{\min}}{F} = \dfrac{1.22\lambda}{D} \quad \Rightarrow \quad d_{\min} = \dfrac{1.22\lambda F}{D}$

Here $\dfrac{D/2}{F} = tan\beta$ [2β is the angle subtended by diameter of objective lens at focus of microscope.)

$\Rightarrow \quad D = 2F\tan\beta \approx 2F\sin\beta$ (for small β) $\quad \therefore \quad d_{\min} = \dfrac{1.22\lambda F}{D} = \dfrac{1.22\,\lambda F}{2F\sin\beta}$

If a medium of refractive index 'n' is placed in between object and objective lens.

$\left(F \to nF\ ;\ \lambda \to \dfrac{\lambda}{n},\quad n \equiv \text{refractive index of medium}\right) \quad \therefore \quad d_{\min} = \dfrac{1.22\lambda}{2n\sin\beta}$

$$\text{Resolving Power} = \frac{1}{d_{min}} = \frac{2n\sin\beta}{1.22\lambda}$$

- Resolving Power can be increased by choosing a medium of higher refractive index.
- Usually an oil having refractive index close to that of the objective glass is used. Such an arrangement is called "oil immersion objective"
- Also, since its not possible to makes *sin* β greater than unity, therefore resolving power of Microscope is determined by wavelength of light used.

IN-CHAPTER EXERCISE-D

1. If the red light is replaced by blue light illuminating the object in a microscope, the resolving power of the microscope :

(A) Decreases **(B)** Increases **(C)** Gets halved **(D)** Remains unchanged

2. Near and far points of healthy human eye respectively are :

(A) 0 and 25 *cm* **(B)** 0 and infinity
(C) 25 *cm* and 100 *cm* **(D)** 25 *cm* and infinity

3. Resolving power of a telescope increases with :

(A) Increased focal length of eye piece **(B)** Increased focal length of objective
(C) Increased aperture of eye piece **(D)** Increased aperture of objective

4. The resolving power of a telescope depends on :

(A) Focal length of eyepiece **(B)** Focal length of objective lens
(C) Length of the telescope **(D)** Diameter of the objective lens

5. A compound microscope has two lenses. The magnifying power of one is 5 and the combined magnifying power is 100. The magnifying power of the other lens is :

(A) 10 **(B)** 20 **(C)** 50 **(D)** 25

6. Long-sighted people who have lost their spectacles can still read a book by looking through a small (3-4 mm) hole in a sheet of paper :

(A) because the fine hole produces an image of the letters at a longer distance
(B) because in doing so the distance of the object is increased
(C) because in doing so the focal length of the eye lens is effectively decreased
(D) because in doing so the focal length of the eye lens is effectively increased

7. Magnification of a microscope can be increased by :

(A) Increasing power of objective and eye lens
(B) Decreasing power of objective and eye lens
(C) Increasing power of objective and decreasing power of eyepiece
(D) Decreasing power of objective and increasing power of eyepiece

8. A doctorprescribes spectacles to a patient with a combination of a convex lens of focal length 40 cm, and concave lens of focal length 25 cm, then the power of spectacles will be:

(A) – 6.5 D **(B)** 1.5 D **(C)** – 1.5 D **(D)** – 8.5 D

9. The diameter of the objective lens of a telescope is 5.0 m and wavelength of light is 6000 Å. The limit of resolution of this telescope is:

(A) 0.15s **(B)** 0.06s **(C)** 0.03s **(D)** 3.03s

10. A telescope has an objective lens of 10 cm diameter and is situtated at a distance of one kilometre from two objects. The minimum distance between these two objects, which can be resolved by the telescope, when the mean wavelength of light is 5000 Å, is of the order of :

(A) 0.5 m **(B)** 5 m **(C)** 5 mm **(D)** 5 cm

11. The length of an astronomical telescope adjusted for parallel light is 100 cm. If the magnification of the telescope is 19, the focal length of two lenses, i.e., objective and eye piece are :

(A) 76 cm and 4 cm **(B)** 95 cm and 5 cm **(C)** 85 cm and 15 cm **(D)** 82 cm and 18 cm

12. A professor reads a greeting card on his 50th birthday with + 2.5 D glasses keeping the card 25 cm away. 10 years later he reads the greeting card with same glass keeping the card 50 cm away. What power glasses should he wear now?

(A) 2 D **(B)** 0.5 D **(C)** 2.25 D **(D)** 4.5 D

13. A simple microscope is rated 5x for a normal relaxed eye. What will be its magnifying power for a farsighted man whose near point is 40 cm?

(A) 5x **(B)** 3x **(C)** 8x **(D)** 13x

14. The focal length of lens of a camera is 10 cm and focussing range 0.6 m out to infinity. find the range of movement necessary between lens and film,

(A) 1 cm **(B)** 2 cm **(C)** 3 cm **(D)** 4 cm

15. The lens of a certain 3 cm film size camera has focal length 50 cm and diameter 10 cm. Its f-number is:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5

16. In previous problem its exposure time is $\frac{1}{400}s.$ Find its exposure time for $\frac{f}{6}.$

(A) $\frac{9}{2500}s$ **(B)** $\frac{3}{250}s$ **(C)** $\frac{9}{200}s$ **(D)** $\frac{1}{100}s$

17. A pinhole camera with 10 cm high film is to be used to take a picture of a man whose height is 2 m. The film is 20 cm from the pinhole. How far should the camera be from the man to include the fulll height of the tree?

(A) 1 m **(B)** 2 m **(C)** 3 m **(D)** 4 m

18. The image of the Sun is focused by achromatic thin convex lens of focal length 75 cm on a screen. The image is seen by eye at least distance of distinct vision from screen. The angular magnification is:

(A) –1 **(B)** –2 **(C)** –3 **(D)** –4

19. The aperture of a telescope is 1.22 m. The wavelength of visible light is 500 nm. The resolving power of telescope is:

(A) 10^6 **(B)** 10^7 **(C)** 2×10^6 **(D)** 4×10^6

20. In previous problem, if distance between the earth and moon is 4×10^8 m. The minimum separation between objects on the moon surface. So that they are just resolved is:

(A) 50 m **(B)** 100 m **(C)** 150 m **(D)** 200 m

ANSWERS TO IN-CHAPTER EXERCISES

A	**1.** B	**2.** C	**3.** A	**4.** A	**5.** B	**6.** B	**7.** B
	8. B	**9.** C	**10.** B	**11.** C	**12.** B	**13.** D	**14.** D
	15. A	**16.** A	**17.** C	**18.** C	**19.** B	**20.** B	**21.** A
	22. D	**23.** C	**24.** C	**25.** A	**26.** A	**27.** B	**28.** C
	29. D	**30.** C	**31.** C	**32.** A	**33.** B	**34.** B	**35.** A
	36. B	**37.** D	**38.** B	**39.** D	**40.** D		
B	**1.** E	**2.** B	**3.** A	**4.** D	**5.** A	**6.** B	**7.** B
	8. A	**9.** A	**10.** C	**11.** B	**12.** B	**13.** A	**14.** D
	15. B	**16.** A	**17.** A	**18.** A	**19.** A	**20.** C	
C	**1.** B	**2.** B	**3.** B	**4.** B	**5.** B	**6.** B	**7.** D
	8. B	**9.** A	**10.** B	**11.** A	**12.** A	**13.** B	**14.** B
	15. A	**16.** C	**17.** B	**18.** A	**19.** A	**20.** B	
D	**1.** B	**2.** D	**3.** D	**4.** D	**5.** B	**6.** C	**7.** A
	8. C	**9.** C	**10.** C	**11.** B	**12.** D	**13.** C	**14.** B
	15. D	**16.** A	**17.** D	**18.** C	**19.** C	**20.** D	